MATH 480
Mathematics Comprehensive
Grading Criteria

Name:_____

Written Communication

Content & Organization			
1. All key elements of the assignments are covered.	5	3	0
2. The content is comprehensive, accurate, and/or persuasive.	5	3	0
3. Major points are stated clearly; are supported by specific details, examples, or analysis; and are organized logically.	5	3	0
4. Details are appropriately supported by research evidence or literary citation using relevant sources.	5	3	0
5. The introduction provides sufficient background on the topic and previews major points, and provides a point of interest for the audience.	5	3	0
Mechanics and Format			
1. The tone is appropriate to the audience, content and assignment.	5	3	0
2. Statements are consistent, complete, clear and concise.	5	3	0
3. Pages are well-constructed with consistently strong and varied sentences.	5	3	0
4. Spelling, grammar and sentence construction helps give clarity to the reader.	5	3	0

MATH 480
Mathematics Comprehensive Content

On the following pages are five questions from each of the courses that are offered as part of the mathematics major at Mayville State University. You must answer correctly at least three items from each set of five that relate to the courses for which you have received credit.

You may write in the exam booklet. Make sure to clearly mark your answer so there is no confusion by the person scoring the exam.

Cross out the parts of the exam that relate to courses for which you have NOT received credit.

All of the exam booklet, scratch paper, and answer sheets must be turned in when you have completed the exam.

MATH 103 = College Algebra

1. The equation of the line that passes through the point (–3,1) and is perpendicular to $y = \frac{1}{2}x + 4$ is:

   a. $y = \frac{1}{2}x + 7$
   b. $y = 2x + 7$
   c. $y = -\frac{1}{2}x + 4$
   d. $y = \frac{1}{2}x + {}^{5}/_{2}$
   e. $y = -2x - 5$

2. If *w* varies directly as the square of *z*, then the graph of this relation is a:

   a. line
   b. ellipse
   c. circle
   d. parabola
   e. hyperbola

3. The degree of $P(x) = x^2(x+3)(x-3)^4$ is:

   a. 6
   b. 7
   c. 8
   d. 16
   e. 9

4. The expression $\frac{\sqrt{2+1}}{\sqrt{2-1}}$ is equivalent to:

   a. 3
   b. 1
   c. $5 + \sqrt{2}$
   d. $3 + 2\sqrt{3}$
   e. $\sqrt{3}$

5. The expression $\frac{a^{-1}+b^{-1}}{a^{-1}}$ is equivalent to:

   a. $\frac{1}{b}$
   b. $\frac{(a+b)}{b}$
   c. $\frac{b}{(a+b)}$
   d. $\frac{a}{(a+b)}$
   e. b

MATH 105 = Trigonometry

6. A function having a period of $\frac{\pi}{2}$ is:

a. $y = ¼ \sin x$
b. $y = 4 \sin 2x$
c. $y = 2 \sin 4x$
d. $y = 4 \sin ¼ x$
e. $y = 2 \sin x$

7. Which one of the following is an identity?

a. $\sin x + \cos x = 1$
b. $\sec x \cdot \csc x = 1$
c. $\sin ½ x = ½ \sin x$
d. $\tan^2 x = \sec^2 x - 1$
e. $\sin x \cdot \tan x = \cos x$

8. In right triangle ΔABC, if $\sin A = ¾$ and $\sin B = ½$, then the ratio of the side length a to side length b is:

a. 3 : 2
b. 8 : 3
c. 4 : 3
d. 3 : 1
e. None of the above

9. The expression $y = 2 \sin 3x$ reaches its maximum value when x equals:

a. 0
b. $\frac{\pi}{6}$
c. $\frac{\pi}{4}$
d. $\frac{2\pi}{3}$
e. None of the above

10. If $f(x) = \cos 3x + \tan 2x$, then $f\left(\frac{\pi}{6}\right)$ equals:

a. $\sqrt{3}$
b. $1 + \sqrt{3}$
c. $\frac{5\sqrt{3}}{6}$
d. $\frac{\sqrt{3}}{3}$
e. $\frac{(3+\sqrt{3})}{3}$

MATH 165 = Calculus I

11. Find $\lim_{x \to 2} \frac{x-2}{x^2-9}$

a. ∞
b. 0
c. 2
d. $-\infty$
e. 4

12. Given the rational function $f(x) = \frac{x-2}{x^2-9}$ which of the following statements is <u>not</u> true about its graph?

a. $x = 3$ and $x = -3$ are vertical asymptotes
b. the point $(-2,0)$ is an x-intercept
c. the x-axis is its horizontal asymptote
d. the point $(0, \frac{2}{9})$ is its y-intercept
e. this function would be discontinuous when $x = 3$

13. Write the equation for the tangent line which can be drawn to the curve $y = x^2$ at the point (2,4):

a. $y = 4x$
b. $y = 2x + 4$
c. $y = 4x - 4$
d. $y = 4x - 16$
e. None of the above

14. If $f(x) = x^2 \sin x$ then find $f'(x)$ [the derivative of f(x)]

a. $2x \cos x$
b. $x^2 \cos x$
c. $x^2 \cos x - 2x \sin x$
d. $-x^2 \cos x + 2x \sin x$
e. $x^2 \cos x + 2x \sin x$

15. Find the area of the region bounded by the curve $y = x^2$ above, the x-axis below and the vertical lines $x = 1$ and $x = 3$ on the sides:

a. $\frac{26}{3}$
b. 18
c. 8
d. $\frac{14}{3}$
e. None of the above

MATH 166 = Calculus II

16. Given the function $f(x) = \frac{1}{\sqrt{4-x^2}}$ find its derivative $f'(x)$ using the chain rule for derivatives:

a. $f'(x) = \sqrt{4-x^2}$
b. $f'(x) = \frac{x}{4-x^2}$
c. $f'(x) = \frac{-x}{\sqrt{4-x^2}}$
d. $f'(x) = \frac{x}{\sqrt{4-x^2}}$
e. None of the above

17. $\int_0^{\pi/2} \cos x \, dx$ is equal to:

a. 0
b. 1
c. $\frac{\pi}{4}$
d. π
e. None of the above

18. Find the volume of the solid of revolution that would be formed if the region bounded by $y = \sqrt{x}$ on top and the x-axis on the bottom between the points x = 0 and x = 4 were to be rotated about the x-axis:

a. $V = 8\pi$
b. $V = \frac{\pi^2}{8}$
c. $V = 16\pi$
d. $V = \frac{16}{3}$
e. None of the above

19. If you integrate $\int x \ln x \, dx$ using the method of integration by parts (let u = ln x and let dv = x dx) you get:

a. $1 + \frac{1}{x}$
b. $x^2 \ln x + \ln x$
c. $x \ln x + x$
d. $\frac{x^2}{2} \ln x - \frac{x^2}{4}$
e. None of the above

20. Which of the following would <u>not</u> be true concerning the ln x function?

a. $\int \ln x \, dx = e^x + c$
b. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
c. $\int_1^4 \frac{1}{x} \, dx = \ln 4$
d. $\ln(e^x) = x$
e. $\ln 1 = 0$

MATH 265 = Calculus III

21. $\int_0^4 \sqrt{16 - x^2}\, dx$ [Hint: $x^2 + y^2 = r^2$ is the formula for a circle]

a. 2π
b. $\frac{\pi}{2}$
c. 4π
d. 8π
e. $4\pi^2$

22. To find $\int \frac{(2x^3-4x-8)}{x^2(x+1)(x^2+4)}\, dx$ using the Method of Partial Fractions, you would set up the following fractions:

a. $\frac{A}{x^2} + \frac{B}{x+1} + \frac{C}{x^2+4}$
b. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{x^2+4}$
c. $\frac{A}{x^2} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$
d. $\frac{A}{x^2} + \frac{B}{x+1} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$
e. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+4}$

23. Find $\lim_{x\to\infty} \frac{lnx}{x}$ using L'Hospital's Rule

a. ∞
b. 0
c. $-\infty$
d. 1
e. ln 2

24. The improper integral $\int_1^\infty \frac{1}{x^2}\, dx$ when evaluated becomes

a. ∞
b. 0
c. $\frac{1}{2}$
d. 1
e. None of the above

25. Which of the following series would be a divergent series?

a. $\Sigma\left(\frac{2}{3}\right)^n$
b. $\Sigma\left(\frac{2^n}{n!}\right)$
c. $\Sigma\frac{(-1)^{n+1}}{n}$
d. $\Sigma\frac{1}{n^2}$
e. $\Sigma\frac{3n+2}{n^2-2}$

MATH 323 = Probability & Statistics

26. The probability of flipping 8 coins and getting exactly 3 heads would be:

    a. $\frac{3}{8}$
    b. $\frac{3}{16}$
    c. $\frac{{}_8C_3}{2^6}$ or $\frac{56}{64}$
    d. ${}_8C_3\,(\frac{1}{2})^8$
    e. None of the above

27. If you were to arrange the letters A, B, C, and D into all of its possible orders (permutations), how many different ways could you arrange these 4 letters?

    a. 4
    b. 4! or 24
    c. ${}_4C_2$ or 6
    d. 12
    e. None of the above

28. The heights of a certain population of adult males are approximately normally-distributed with a mean of $\mu = 71$ inches and a standard deviation of $\sigma = 3$ inches. Then the probability that an adult male selected from this population will have a height between 65 and 77 inches will be approximately:

    a. 0.50
    b. 0.40
    c. 0.68
    d. 0.95
    e. 0.99

29. The best statistic to use as an estimate for μ (the mean of a normal distribution) would be:

    a. the sample mean $\bar{x}$
    b. the sample size n
    c. the sample median
    d. the sample standard deviation σ
    e. the sample correlation coefficient r

30. When testing an hypothesis about the value of a parameter from some population at the $\alpha = 0.05$ level of significance there would be:

    a. a 5% chance of accepting the null hypothesis
    b. a 5% chance of accepting the null hypothesis when it is false
    c. a 5% chance of rejecting the null hypothesis when it is false
    d. a 5% chance of rejecting the null hypothesis when it is true
    e. None of the above

MATH 389 = Modern Geometry

31. For an equilateral triangle, the following coincide to the same point:

a. orthocenter, incenter, midpoint, centroid
b. orthocenter, incenter, circumcenter, excenter
c. orthocenter, incenter, circumcenter, centroid
d. orthocenter, incenter, centroid, bisector

32 In non-Euclidean geometry, which of Euclid's postulates is <u>not</u> held to be true?

a. First postulate, dealing with existence of two points
b. Second postulate, dealing with two points define a line
c. Third postulate, dealing with each line is on two points
d. Fourth postulate, dealing with vertical angles
e. Fifth postulate, dealing with parallel lines

33. The medians of a triangle meet at a point called the:

a. orthocenter
b. incenter
c. centroid
d. circumcenter
e. excenter

34. The three ways to transform an object are:

a. rotation, glide, translation
b. flip, rotation, spin
c. slide, rotation, spin
d. rotation, reflection, translation
e. reflection, rotation, flip

35. All the platonic solids, which are regular polyhedra, are:

a. octahedron, cube
b. octahedron, cube, icosahedron
c. octahedron, tetrahedron, cube, icosahedron
d. dodecahedron, pentahedron, cube, tetrahedron, octahedron
e. octahedron, tetrahedron, cube, dodecahedron, icosahedron

MATH 412 = Differential Equations

36. Which of the following Differential Equations would be a linear, first order differential equation?

a. $2y' + \frac{y}{4} = e^x$
b. $y'' + y' + y = 0$
c. $2y' - 3\sqrt{y} = x$
d. $y' + x y^2 = 4$
e. None of these would be classified as a linear, first order differential equation

37. If you solve the differential equation, $\frac{dy}{dx} = \frac{2x}{3y^2}$, the general solution could be written as:

a. $y^2 = x + c$
b. $y^3 = x^2 + c$
c. $x^2 = y^3 + c$
d. $y = \sqrt{\frac{2x}{3}} + c$
e. None of these would be a general solution

38. Solve the initial value problem consisting of the differential equation: $y'' = 12x - 6$ and the two initial conditions: $y'(0) = 4$ and $y(0) = 5$ for its particular solution.

a. $y = 6x^2 - 6x$
b. $y = x^3 - 3x^2 + 4x + 5$
c. $y = 2x^3 - 2x^2 + 4x + 5$
d. $y = 2x^3 - 3x^2 + 4x + 5$
e. $y = 2x^3 \cdot 3x^2$

39. The equation for the curve passing through the point (0,1) and whose slope at any point (x,y) on the curve is equal to its y coordinate would be:

a. $y = e^x$
b. $y = e^{x^2}$
c. $y = x$
d. $y = x + 1$
e. $y = e^x + 1$

40. The general solution to this homogeneous differential equation: $y'' - 2y' - 15y = 0$ would be:

a. $y = c_1x^3 + c_2x^{-5}$
b. $y = c_1e^{2x} + c_2e^{-5x}$
c. $y = c_1e^{3x} + c_2e^{-5x}$
d. $y = c_1e^{2x} + c_2e^{-2x}$
e. None of these would be a general solution

MATH 420 = History & Philosophy of Mathematics

41. These two mathematicians each discovered calculus in the 1600s working independently.

    a. VanGogh and Saccheri
    b. Euclid and Beethoven
    c. Germain and Gauss
    d. Newton and Leibniz
    e. Gauss and Euler

42. This man from Syracuse, Sicily is known for his buoyancy principle, his law of levers, and engineering weapons of war was:

    a. Saccheri
    b. Euclid
    c. Archimedes
    d. Thales
    e. Plato

43. This Greek man wrote a series of 13 books called *Elements*, which is considered the most successful textbook ever written.

    a. Archimedes
    b. Euclid
    c. Thales
    d. Pythagoras
    e. Eudoxus

44. These mathematicians were both prolific writers, were famous for their work with number theory, and one produced many works after becoming blind.

    a. VanGogh and Saccheri
    b. Euclid and Beethoven
    c. Germain and Gauss
    d. Newton and Leibniz
    e. Gauss and Euler

45. The father of analytic geometry is:

    a. Gauss
    b. Abel
    c. Galois
    d. Descartes
    e. Cantor

MATH 435 = Theory of Numbers

46. Which of the following true statements is known as the "Fundamental Theorem of Arithmetic?"
    a. There are an infinite number of primes
    b. If p is a prime number and p divides the product bc then p must divide into b or p must divide into c
    c. If p is a prime number then $\sqrt{p}$ would be an irrational number
    d. The only value of n for which the equations $x^n + y^n = z^n$ would have a solution in the set of integers is n = 2. Thus $x^2 + y^2 = z^2$ has solutions but $x^3 + y^3 = z^3$, $x^4 + y^4 = z^4$, etc would have none
    e. Every positive integer n > 1 can be expressed as a product of primes and this expression would be unique except for the order in which the factors occur

47. The symbol a | b means that a divides b. By definition we have a | b if there exists an integer q such that aq = b. Which of the following statements about this "divides" relation would <u>not always</u> be true?
    a. If a | b then a is a factor of b
    b. If a | b and b | c then a | c
    c. If a | b and a|c then a | (b+c)
    d If a | b then b is a multiple of a
    e. None of the above

48. If you convert the base six numeral 342_6 into its equivalent base 8 numeral, you would get:
    a. 243_8
    b. 134_8
    c. 206_8
    d. 1134_8
    e. None of the above

49. Which of the following expressions would <u>not</u> be appropriate summations to represent the sum of this series: 28 + 31 + 34 + 37 + 40 + 43?
    a. $\sum_{i=9}^{14} 3i + 1$
    b. $\sum_{i=10}^{15} (3i - 2)$
    c. $\sum_{i=1}^{14} (3i + 1) - \sum_{i=1}^{8} (3i + 1)$
    d. $\sum_{i=25}^{40} (i + 3)$
    e. $\sum_{i=8}^{13} 3i + 4$

50. Which of the following Linear Diophantine Equations would <u>not</u> have a solution in the set of integers?
    a. 5x + 10y = 25
    b. 8x + 6y = 40
    c. 7x + 14y = 350
    d. 5x + 9y = 24
    e. 2x + 4y = 17

MATH 443 = Algebraic Structures

51. Let S be the set of integers modulo n where n is a positive integer. With the operations of addition and multiplication modulo n, S forms a field if and only if n is:

a. three
b. seven
c. even
d. odd
e. prime

52. If a and b are non-zero integers and p is prime such that p divides the product of a and b, then:

a. p divides a
b. p divides b
c. p divides a and p divides b
d. p divides a or p divides b
e. p does not divide a

53. The determinant of the product $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$ is:

a. –1
b. –2
c. 1
d. 0
e. Undefined

54. Which of the following is a basis for the vector space of all ordered triples of real numbers?

a. {(1,2,3) , (–1,1,2)}
b. {(1,1,0) , (1,0,1) , (0,1,1)}
c. {(1,1,1) , (2,–1,1) , (1,–5,–1)}
d. {(1,1,1) , (2,1,1) , (2,2,2)}
e. {(1,0,0) , (1,1,1) , (2,1,–1) , (0,0,1)}

55. Evaluate the following determinant: $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$

a. 1
b. –2
c. 2
d. –4
e. 4