Students took the final exam using MyMathLab. Here is a Word document with images of the types of problems they did.

The Word document does not reflect that the exam had screen reader capability. Be sure to indicate that so that we do not lose points for not being compliant with the correct form.

1. Find a basis for the null space of the matrix given below.

$$\begin{bmatrix} 1 & 1 & -4 & 2 & 12 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -6 & 0 & 18 \end{bmatrix}$$

A basis for the null space is { [ ] }.
(Use a comma to separate answers as needed.)

2. Let the matrix below act on $\mathbb{C}^2$. Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$.

$$\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$$

The eigenvalues of $\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$ are [ ].
(Type an exact answer, using radicals and i as needed. Use a comma to separate answers as needed.)

A basis vector for the eigenspace corresponding to the eigenvalue $a + bi$, where $b > 0$, is [ ].
(Type an exact answer, using radicals and i as needed.)

A basis vector for the eigenspace corresponding to the eigenvalue $a - bi$ where $b > 0$, is [ ].
(Type an exact answer, using radicals and i as needed.)

3. Find the characteristic polynomial of the matrix, using either a cofactor expansion or the special formula for 3×3 determinants. [Note: Finding the characteristic polynomial of a 3×3 matrix is not easy to do with just row operations, because the variable λ is involved.]

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & -2 \\ 0 & 7 & 0 \end{bmatrix}$$

The characteristic polynomial is ______.
(Type an expression using λ as the variable.)

4. Compute $\left(\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{x} \cdot \mathbf{x}}\right)\mathbf{x}$ using the vectors $\mathbf{w} = \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$.

$\left(\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{x} \cdot \mathbf{x}}\right)\mathbf{x} =$ ______
(Simplify your answer. Type an integer or simplified fraction for each matrix element.)

5. Find the distance between $\mathbf{x} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -4 \\ -9 \end{bmatrix}$.

The distance between **x** and **y** is ______.
(Type an exact answer, using radicals as needed.)

6. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$. Then express **x** as a linear combination of the **u**'s.

$$\mathbf{u}_1 = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \text{ and } \mathbf{x} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Which of the following criteria are necessary for a set of vectors to be an orthogonal basis for a subspace W of $\mathbb{R}^n$? Select all that apply.

- ☐ **A.** The vectors must form an orthogonal set.
- ☐ **B.** The distance between any pair of distinct vectors must be constant.
- ☐ **C.** The vectors must all have a length of 1.
- ☐ **D.** The vectors must span W.

Which theorem could help prove one of these criteria from another?

- ○ **A.** If $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ is a basis in $\mathbb{R}^p$, then the members of S span $\mathbb{R}^p$ and hence form an orthogonal set.

- B. If $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ is an orthogonal set of nonzero vectors in $\mathbb{R}^n$, then S is linearly independent and hence is a basis for the subspace spanned by S.
- C. If $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ and each $\mathbf{u}_i$ has length 1, then S is an orthogonal set and hence is a basis for the subspace spanned by S.
- D. If $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ and the distance between any pair of distinct vectors is constant, then the vectors are evenly spaced and hence form an orthogonal set.

Which calculations should be performed next?
(Simplify your answers.)

A.	B.	C.
$\mathbf{u}_1 \cdot \mathbf{u}_1 =$ ____	$\mathbf{u}_1 - \mathbf{u}_2 =$ ____	$\mathbf{u}_1 \cdot \mathbf{u}_2 =$ ____
$\mathbf{u}_2 \cdot \mathbf{u}_2 =$ ____	$\mathbf{u}_1 - \mathbf{u}_3 =$ ____	$\mathbf{u}_1 \cdot \mathbf{u}_3 =$ ____
$\mathbf{u}_3 \cdot \mathbf{u}_3 =$ ____	$\mathbf{u}_2 - \mathbf{u}_3 =$ ____	$\mathbf{u}_2 \cdot \mathbf{u}_3 =$ ____

How do these calculations show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$?

Since each (1) ________ is ____, the vectors (2) ________ From the theorem above, this proves that the vectors are also (3) ________

Express **x** as a linear combination of the **u**'s.

$\mathbf{x} =$ ____ $\mathbf{u}_1$ + ____ $\mathbf{u}_2$ + ____ $\mathbf{u}_3$
(Use integers or fractions for any numbers in the equation.)

(1)
- difference
- inner product

(2)
- form an orthogonal set.
- all have length 1.
- are uniformly spaced.
- form a basis.

(3)
- an orthogonal set.
- of length 1.
- evenly spaced.
- a basis.

7. Compute the adjugate of the given matrix, and then use the Inverse Formula to give the inverse of the matrix.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 3 & 3 \end{bmatrix}$$

The adjugate of the given matrix is adj A= ☐.
(Type an integer or simplified fraction for each matrix element.)

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The inverse matrix is A^{-1} = ☐.
  (Type an integer or simplified fraction for each matrix element.)
- ○ **B.** The matrix is not invertible.

8. List the eigenvalues of A. The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the composition of a rotation and a scaling. Give the angle φ of the rotation, where $-\pi < \varphi \le \pi$, and give the scale factor r.

$$A = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & -0.1 \end{bmatrix}$$

The eigenvalues of A are λ = ☐.
(Use a comma to separate answers as needed. Type an exact answer, using radicals and i as needed.)

φ = ☐
(Type an exact answer, using π as needed.)

r = ☐
(Type an exact answer, using radicals as needed.)

9. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 2 & 3 & -5 & 1 \\ 0 & -3 & 8 & -5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & -5 & 1 \\ 0 & -3 & 8 & -5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 5 \end{vmatrix} = \square$$ (Simplify your answer.)

10. Solve the system.

$$\begin{aligned} x_1 \qquad\quad - 6x_3 &= 23 \\ 2x_1 + 4x_2 + x_3 &= 31 \\ 2x_2 + 3x_3 &= 3 \end{aligned}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- ○ **A.** The unique solution of the system is (x_1,x_2,x_3) = (☐,☐,☐).
  (Type integers or simplified fractions.)
- ○ **B.** The system has infinitely many solutions.
- ○ **C.** The system has no solution.

11. Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 5 & -7 & 2 & 0 \\ 10 & -14 & 4 & 0 \\ 20 & -28 & 8 & 0 \end{bmatrix}$$

Choose the correct answer below.

- **A.** $\begin{cases} x_1 = -5x_2 \\ x_2 = 7x_3 \\ x_3 \text{ is free} \end{cases}$
- **B.** $\begin{cases} x_1 = \frac{7}{5}x_2 - \frac{2}{5}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$
- **C.** $\begin{cases} x_1 = 5 \\ x_2 = -7 \\ x_3 = 2 \end{cases}$
- **D.** The system has no solutions.

12. Determine whether the statement below is true or false. Justify the answer.

The echelon form of a matrix is unique.

Choose the correct answer below.

- **A.** The statement is false. Neither the echelon form nor the reduced echelon form of a matrix are unique. They depend on the row operations performed.
- **B.** The statement is true. Both the echelon form and the reduced echelon form of a matrix are unique. They are the same regardless of the chosen row operations.
- **C.** The statement is false. The echelon form of a matrix is not unique, but the reduced echelon form is unique.
- **D.** The statement is true. The echelon form of a matrix is always unique, but the reduced echelon form of a matrix might not be unique.

13. Compute the product AB by the definition of the product of matrices, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and by the row-column rule for computing AB.

$$A = \begin{bmatrix} -1 & 4 \\ 1 & 3 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$$

Set up the product $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$A\mathbf{b}_1$ = [ ] [ ]

(Use one answer box for A and use the other answer box for $\mathbf{b}_1$.)

Calculate $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$A\mathbf{b}_1$ = [ ]

(Type an integer or decimal for each matrix element.)

Set up the product $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$A\mathbf{b}_2$ = [ ] [ ]

(Use one answer box for A and use the other answer box for $\mathbf{b}_2$.)

Calculate $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$A\mathbf{b}_2$ = [ ]

(Type an integer or decimal for each matrix element.)

Determine the numerical expression for the first entry in the first column of AB using the row-column rule. Choose the correct answer below.

- A. $-1(4) + 4(-3)$
- B. $((-1) + (4)) \cdot ((4) + (-3)$
- C. $((-1) - (4)) \cdot ((4) - (-3)$
- D. $-1(4) - 4(-3)$

Determine the product AB.

AB = [ ]

(Use integers or decimals for any numbers in the expression.)

14. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 7 \\ 1 & 7 & 6 \end{bmatrix}$ and $D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Compute AD and DA. Explain how the columns or rows of A change when A is multiplied by D on the right or on the left. Find a 3×3 matrix B, not the identity matrix or zero matrix, such that AB = BA.

Compute AD.

AD = ▭

Compute DA.

DA = ▭

Explain how the columns or rows of A change when A is multiplied by D on the right or on the left. Choose the correct answer below.

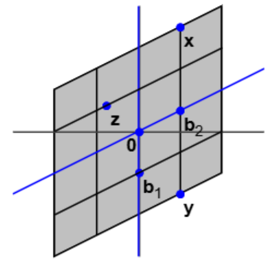
- **A.** Both right-multiplication (that is, multiplication on the right) and left-multiplication by the diagonal matrix D multiplies each column entry of A by the corresponding diagonal entry of D.
- **B.** Right-multiplication (that is, multiplication on the right) by the diagonal matrix D multiplies each row of A by the corresponding diagonal entry of D. Left-multiplication by D multiplies each column of A by the corresponding diagonal entry of D.
- **C.** Both right-multiplication (that is, multiplication on the right) and left-multiplication by the diagonal matrix D multiplies each row entry of A by the corresponding diagonal entry of D.
- **D.** Right-multiplication (that is, multiplication on the right) by the diagonal matrix D multiplies each column of A by the corresponding diagonal entry of D. Left-multiplication by D multiplies each row of A by the corresponding diagonal entry of D.

Find a 3×3 matrix B, not the identity matrix or zero matrix, such that AB = BA. Choose the correct answer below.

Find a 3×3 matrix B, not the identity matrix or zero matrix, such that AB = BA. Choose the correct answer below.

- **A.** There is only one unique solution, B = ▭.
  (Simplify your answers.)
- **B.** There are infinitely many solutions. Any multiple of I_3 will satisfy the expression.
- **C.** There does not exist a matrix, B, that will satisfy the expression.

15. Let $\mathbf{b}_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, $\mathbf{z} = \begin{bmatrix} -1.5 \\ 1.25 \end{bmatrix}$ and $B = \{\mathbf{b}_1, \mathbf{b}_2\}$. Use the figure to estimate $[\mathbf{x}]_B$, $[\mathbf{y}]_B$, and $[\mathbf{z}]_B$. Confirm your estimates of $[\mathbf{y}]_B$ and $[\mathbf{z}]_B$ by using them and $\{\mathbf{b}_1, \mathbf{b}_2\}$ to compute $\mathbf{y}$ and $\mathbf{z}$.



Use the figure to estimate $[\mathbf{x}]_B$. Choose the correct answer below.

- A. $[\mathbf{x}]_B = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$
- B. $[\mathbf{x}]_B = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$
- C. $[\mathbf{x}]_B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
- D. $[\mathbf{x}]_B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Use the figure to estimate $[\mathbf{y}]_B$. Choose the correct answer below.

- A. $[\mathbf{y}]_B = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$
- B. $[\mathbf{y}]_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- C. $[\mathbf{y}]_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
- D. $[\mathbf{y}]_B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

Use the figure to estimate $[\mathbf{z}]_B$. Choose the correct answer below.

- A. $[\mathbf{z}]_B = \begin{bmatrix} \frac{3}{4} \\ 1 \end{bmatrix}$
- B. $[\mathbf{z}]_B = \begin{bmatrix} -\frac{3}{4} \\ -1 \end{bmatrix}$
- C. $[\mathbf{z}]_B = \begin{bmatrix} 1 \\ \frac{3}{4} \end{bmatrix}$
- D. $[\mathbf{z}]_B = \begin{bmatrix} -1 \\ -\frac{3}{4} \end{bmatrix}$

16. What is the rank of a 4×6 matrix whose null space is three dimensional?

rank A = ______

17. If the rank of a 6×9 matrix A is 3, what is the dimension of the solution space $A\mathbf{x} = \mathbf{0}$?

The dimension of the solution space is ______.

*18. Use Cramer's rule to solve the system.

$$\begin{aligned} 7x + 2y &= 33 \\ 5x - 3y &= -3 \end{aligned}$$

Write the fractions using Cramer's Rule in the form of determinants.

$x = \dfrac{\square}{\square}$ $y = \dfrac{\square}{\square}$

The solution set is $\{\square\}$.

(Simplify your answer. Type an ordered pair, using integers or fractions.)